

Resistance Jumps and Hysteresis in Ferromagnetic Wires

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A phenomenological theory based on the principle of minimum heat generation is presented for the recently discovered exotic electric transport in ferromagnetic wires with a domain wall. We provide an unified explanation of the negative and positive jumps and of the hysteresis loops observed in the magnetoresistance experiments. Our argument is based on a microscopic calculation which shows that the presence of a domain wall sometimes enhances the transmission probability through an impurity potential, as well as the phenomenological principle of minimum heat generation.

The Ohm's law, which states that the measured voltage is proportional to the current flow, has been observed in many conducting materials. It is usually regarded that Ohmic transport can be explained by means of a suitable linear response theory. Non-Ohmic transports, where the voltage has a nonlinear dependence on the current, are also observed in many conductors. Unlike Ohmic transport, non-Ohmic transport usually results from complicated nonlinear responses of a microscopic state which takes place when a finite current flows through the material.

Recently, in a series of experiments in metallic wires containing ferromagnetic domain walls [1–4], exotic non-Ohmic transport phenomena were observed. The transport properties exhibit a delicate dependence on the external magnetic field which controls the motion of the domain walls. Most remarkably, the non-Ohmic resistance (defined simply as the ratio between the voltage and the current) shows an abrupt negative jump and a hysteresis loop when the magnetic field is varied. It is believed that these peculiar behaviors are caused by the motion of the ferromagnetic domain wall(s).

Although some microscopic mechanisms were proposed [4–6] in order to explain these phenomena, no satisfactory unified picture exists at present for the whole phenomena, and there still remain many issues (e.g., the magnitudes of the resistance jumps, and the shape of the hysteresis loop) to be explained. Clearly the difficulty comes from the nonlinearity and the complexity. The standard linear response theory is obviously useless in dealing with non-Ohmic transport. We also expect that various effects such as the crystal anisotropy, the Lorentz force, and the scattering by various scatterers (e.g., impurities, domain walls, localized spins, and the surface of the sample) are entangled with each other in a complex

manner thus producing the observed transport properties. It seems unlikely to us that one would be able to treat these effects separately. A phenomenological approach which deals with overall properties of the system as a whole seems most suitable in studying such complex nonlinear phenomena.

In the present Letter we propose a phenomenological explanation for the non-Ohmic nature, the negative and positive jumps, and the hysteresis loops, in the magnetoresistance experiments for the ferromagnetic wires. Our basic strategy is to apply the principle of minimum heat generation. The principle states that, when a given amount of electric current goes through a sample, the local currents distribute themselves so as to make the total heat generation rate as small as possible. (See, e.g., Chap. 19 of Ref. [7].) As is well known, when the resistance (or the resistance distribution) of the sample is independent of the current, the principle leads to Ohmic (or the Kirchhoff's) law. When the resistance depends on the current, then the principle may lead to a nonlinear voltage-current relation.

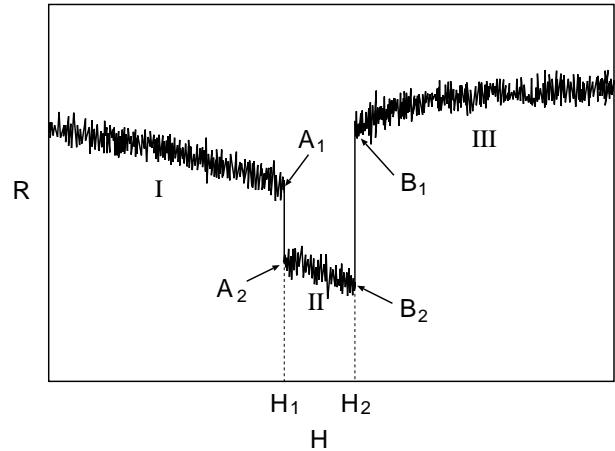


FIG. 1. A typical resistance curve R as a function of an external magnetic field H . The curve consists of three stages and two resistance jumps.

Figure 1 schematically shows a typical experimental result [2] of the magnetoresistance R of a ferromagnetic wire under a uniform magnetic field H parallel to the current. In the initial state, the wire is set in a strong magnetic field (which is taken to be negative for the following convenience) and all the domain walls are swept out of the wire. The magnetic field then is increased from the initial negative value and swept towards posi-

tive values, in order to create and control a single domain wall in the wire. In Stage I in Fig. 1, the resistance R decreases with fluctuations as the magnetic field H is increased. It has been confirmed experimentally that there is no domain wall in the wire at this stage. At the magnetic field H_1 , the resistance R abruptly jumps from the point A_1 to A_2 , and this negative jump of the resistance R is accompanied with an abrupt appearance of a single ferromagnetic domain wall in the wire. In Stage II, the resistance R decreases with fluctuations as the magnetic field H is increased, and it was observed that the single domain wall remains in the wire. At the magnetic field H_2 , the resistance R abruptly jumps from the point B_2 to B_1 , and this positive jump of the resistance R is accompanied with an abrupt disappearance of the domain wall. After this resistance jump, the magnetization of the wire is fairly saturated in the positive direction parallel to the magnetic field. In the final Stage III, the resistance R increases with fluctuations as the magnetic field H is increased, and no domain wall is observed in the wire. This resistance curve R is not reversible for the reversal process where the magnetic field H is gradually decreased. Hong and Giordano [1] pointed out that this hysteretic behavior of the resistance R is closely related to the hysteresis of the magnetization process and to the existence of a domain wall in the ferromagnet.

In order to understand this behavior, consider a metallic wire with impurities and a ferromagnetic domain wall. Though the impurities potential is fixed, there remain many degrees of freedom which affect the transport properties, and it will be shown that they indeed lead to non-Ohmic characters. As mentioned above, these degrees of freedom include the position of a domain wall and the structure of magnetic domains. According to the principle of minimum heat generation, they distribute themselves in the wire so as to minimize the total heat generation rate.

We begin with a simple situation where there is only one domain wall in the wire. When there is no trapping potential of impurities, the domain wall freely moves in the wire. When there is a fixed impurities potential, which position does the domain wall favor? If the domain wall which entered the wire is abruptly trapped at a special position that minimizes the scattering amplitude of the conduction electrons from the impurities potential, then a negative resistance jump as in Fig. 1 is expected to take place. In order to justify this picture, we use the effective one-dimensional Schrödinger equation [8] with the single domain wall with the center at $x = x_0$,

$$\begin{aligned} & \left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{J\Delta}{4} \tanh\left(\frac{x-x_0}{\lambda}\right) \sigma_x + V_{\text{imp}}(x) \right] \psi(x) \\ &= \frac{J}{4} \operatorname{sech}\left(\frac{x-x_0}{\lambda}\right) \sigma_z \psi(x) + E \psi(x) \end{aligned} \quad (1)$$

for the conduction electron with an energy E , the mass

m , and with the wavefunction,

$$\psi(x) = \begin{pmatrix} \psi_\uparrow(x) \\ \psi_\downarrow(x) \end{pmatrix}; \sigma_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ and } \sigma_z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

The spin $s = \uparrow, \downarrow$ of the electron interacts with the ferromagnetic domain wall through the effective potentials $J\Delta \tanh[(x - x_0)/\lambda]\sigma_x/4$ and $-J\operatorname{sech}[(x - x_0)/\lambda]\sigma_z/4$, where J is the Hund coupling, Δ the anisotropy and λ the width of the domain wall. The domain wall should be realized on the quantum ferromagnetic Heisenberg model, and the width λ is determined by the exchange anisotropy of the Heisenberg model [9]. See Ref. [10] for details of the microscopic derivation of the effective Schrödinger equation (1). We take the impurity potential $V_{\text{imp}}(x)$ to be a rectangular barrier as

$$V_{\text{imp}}(x) = \begin{cases} V_0, & \text{if } |x| \leq \frac{w}{2} \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

with the width w and the height $V_0 > 0$. As we shall see below, details of the impurity potential do not affect our phenomenological argument to understand the experimental results. We assume that the potential $V_{\text{imp}}(x)$ is fixed (i.e., quenched). Although the position x_0 of the domain wall varies to realize the minimum heat generation, we assume that the position x_0 can be treated as a fixed parameter in Eq. (1) because the motion of the domain wall is much slower than that of conduction electrons.

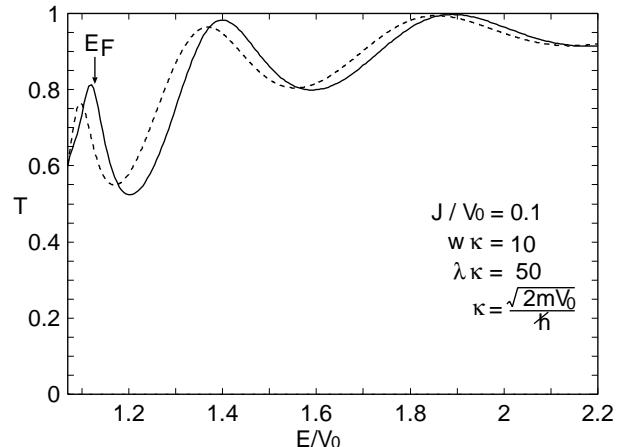


FIG. 2. Transmission probabilities T as a function of energy E of the conduction electron through the rectangular potential without (dashed line) and with (solid line) the single domain wall sitting right above the potential.

We numerically solved the above Schrödinger equation (1) for the case $x_0 = 0$ (i.e., the domain wall sits right above the impurity potential), and obtained the transmission probabilities $T = (T_\uparrow + T_\downarrow)/2$ for a single electron. Here T_s is the transmission probability for the incident plane wavefunction with the polarized spin s , and for simplicity, we have assumed that the spin up and down electrons near the Fermi level E_F equally contribute to

the transport. This assumption does not affect our conclusion below because T_{\downarrow} exhibits a very similar behavior to T_{\uparrow} , where we varied the energy E in our numerical results. Figure 2 shows the calculated transmission probabilities through the rectangular impurity potential with and without the domain wall.

In the case with the simple rectangle barrier and without domain walls, the transmission probability T (dashed line in Fig. 2) oscillates between the minimum and the maximum values as the energy E of the electron increases. When the domain wall is coupled to the impurity potential, the same quantity (solid line in Fig. 2) shows a similar oscillation but with a different phase. As a consequence, we find that there are some ranges of energy E in which the transmission probability T with the domain wall is strictly greater than that without a domain wall. In the generic case for the position of the domain wall, we assume that the distance $|x_0|$ between the domain wall and the impurity is bounded as $|x_0| \leq \bar{x}$ with a positive constant \bar{x} , since actual impurities are almost homogeneously distributed in a wire. In other words, there exists at least one impurity within a region $[a, b]$ with $|b - a| = 2\bar{x}$. Under this assumption, we obtain, from the results in Fig. 2, the following: At least for electrons with energy E in one of the above ranges, there exists an optimal position for the domain wall at which the transmission probability T is maximized and hence the heat generation is minimized [11].

Consequently we find from our numerical results for the position $x_0 = 0$ of the domain wall that the presence of a domain wall enhances the transmission probability through an impurity potential in the wire if the Fermi energy E_F of the conduction electrons is in the above energy range. In a realistic situation, the potential for most impurities, however, is not of a simple rectangular form. But, since real impurities are randomly distributed in a wire, we expect that a potential consisting of many rectangular potentials having various widths and various heights yields the same effect as that of a realistic impurities potential. Furthermore, from the above result for a single rectangular potential, we find that the transmission probability through the wire is enhanced by a single domain wall trapped at a special rectangular potential that satisfies the above energy condition. Combining these observations with the principle of minimum heat generation, we conclude that a single domain wall entered in the wire is trapped at the special position that minimizes the scattering amplitude of the electron at the Fermi level E_F . This explains the negative resistance jump from the point A_1 to A_2 in Fig. 1. In this description, the magnitude ΔR of the resistance jump is evaluated as $\Delta R \approx \Delta T R \lambda / L$ with the length L of the wire and the increase ΔT of the transmission probability due to the domain wall. Substituting the experimental data [2], $\Delta R \approx 0.005 \Omega$, $R \approx 41 \Omega$, $\lambda = 20 \text{ nm}$ and $L = 20 \mu\text{m}$, into this relation, we have $\Delta T \approx 0.12$ which

is consistent with our numerical result $\Delta T \approx 0.15$ which is read from Fig. 2.

In order to explain Stage I in Fig. 1, consider first a simple situation that there is only one small island of localized up spins in the background sea of localized down spins. Note that, if there is no impurity in the wire, the position of the island is not determined by the energetics alone because of the translational invariance of the system. When many impurities exist in the wire, the position of the island is determined by the principle of minimum heat generation, so that the scattering of the electrons at the Fermi level E_F by the impurities potential is suppressed by the presence of the island. Here the mechanism of the suppression for the scattering is essentially the same as in the above case with a single domain wall. Having this result in mind, let us consider the situation in Stage I in Fig. 1. In this stage, there appear many islands of up spins as the external magnetic field H is increased, as schematically shown in Fig. 3.

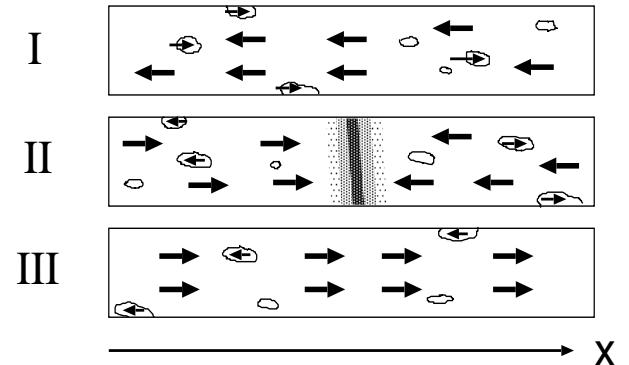


FIG. 3. Typical snapshots of the magnetization profiles in the three stages in the resistance curve R .

The spatial configuration of these islands is also determined by the principle of minimum heat generation so that each island suppresses the scattering of the conduction electrons by an impurity, by combining itself with the impurity. In consequence, the resistance R of the wire decreases as the number of the islands increases. Since the number of the islands increases as the magnetic field H is increased, we recover the observed magnetic field dependence of the resistance R . Moreover, since the total size of the islands is roughly proportional to the total magnetization of the wire, we see that, roughly speaking, the resistance R in Stage I must be a function of the total magnetization. This clearly explains why the resistance curve R shows hysteretic behaviors as those found in the magnetization curve of the ferromagnet. For the same reason, we expect similar hysteretic behaviors in Stages II and III.

In the same way as in Stage I, we can explain Stage II. Namely the increase of the number of up spin islands in the sea of down spins decreases the resistance R , while a single domain wall trapped at a special position also sup-

presses the scattering of the electrons by the impurities potential.

With the abrupt resistance jump from the point B_2 to B_1 , the domain wall and many islands of up spins disappear from the wire. Since these degrees of freedom contributed in decreasing the resistance R , their disappearance leads to an abrupt positive resistance jump as in Fig. 1. After this abrupt change, i.e., in Stage III, the magnetization of the wire is nearly saturated. In other words, there sparsely exist down spin islands in the background sea of up spins, as schematically shown in Stage III of Fig. 3. As the magnetic field H is further increased, the down spin islands disappear or become smaller, and finally all of them disappear from the wire. Since these degrees of freedom also had an effect of decreasing the resistance R , their gradual disappearance leads to an increase of the resistance R observed in Stage III.

Let us conclude by making three remarks. When a single domain wall enters in the wire at the magnetic field H_1 , most of up spin islands are expected to be swept out of the left-hand side region of the domain wall. (See Stages I and II of Fig. 3.) Since these up spin islands contributed in decreasing the resistance R , their abrupt disappearance has an effect of increasing the resistance R . If this positive contribution is larger than the negative one from the entrance of the domain wall, then the resistance jump becomes positive. In the experiment of Ref. [2], such positive resistance jumps were indeed observed.

In an experiment of a ferromagnetic wire, one can make an artificial trapping potential to trap a single domain wall at a fixed position in the wire, by varying locally the width or the shape of the wire. When the trap works well, a negative or positive resistance jump is always expected.

In our explanation of the present phenomena, we assumed that a domain wall and an island in a magnetic domain structure can freely move and flexibly combine with an impurity to reduce the resistance. However, this assumption is not necessarily valid for a general ferromagnetic wire. In fact, a positive resistance contribution from a magnetic domain wall or a magnetic domain structure was observed for some materials [12] which have a rigid magnetic domain structure.

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